

An Anticipatory Terminal Iterative Learning Control Approach with Applications to Constrained Batch Processes^{*}

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Abstract This work presents an anticipatory terminal iterative learning control scheme for a class of batch processes, where only the final system output is measurable and the control input is constant in each operations. The proposed approach works well with input constraints provided that the desired control input with respect to the desired trajectory is within the saturation bound. The tracking error convergence is established with rigorous mathematical analysis. Simulation results are provided to show the effectiveness of the proposed approach.

Keywords terminal iterative learning control, batch-to-batch processes, input saturation, convergence analysis

1 INTRODUCTION

Batch-to-batch processes are commonly found in many industrial sectors [1] and many control approaches of batch processes have been proposed, such as nonlinear optimization and control [1–3], hierarchical supervisory control [4], and so on. It is worth pointing out that the main features of a batch process can be summarized as: (1) it is a high nonlinear time-varying system without steady state operating condition, (2) it is difficult to gain the perfect model, and (3) the intermediate system state and output are not available. Thus it is difficult to obtain desired product properties using the existed control methods [1–4].

It is noted that most of the batch processes execute the repeated control tasks in a finite time interval. This repeatability can be utilized to improve system control performance by iterative learning control (ILC) methods [5–7]. However, the conventional ILC that drives to follow a desired output trajectory in a given time interval through learning iteratively, is not applicable to such a control task because the exact measurements of the intermediate system state and output are impossible [8].

To overcome these difficulties, terminal iterative learning control (TILC) [8] is developed to use the terminal point only at the end of every run and has been applied to many practical applications [8–15]. Its tracking objective is the terminal output rather than the whole output trajectory, *i.e.*, TILC aims to solve the control problem wherein only the terminal output error is concerned. Ref. [8] presents a higher-order TILC for linear systems using a basis function matrix. Xu and Huang [16] studied TILC method for general dissipative systems. Some optimal terminal ILCs were explored in [17, 18]. Xiong and Zhang [9] proposed a

neural network model based batch-to-batch optimal TILC.

From the practical viewpoint, most of the batch systems are under some hard constraints due to physical limitations or performance requirements. Hence the controller design must take these constraints into account. Although some anticipatory ILC schemes with input constraints [19–21] were proposed recently, most of the current terminal ILC researches are still based on assumed unconstrained systems. Only paper [17] has been reported regarding the constrained cases, while its limitation lies in the requirement of the knowledge of a perfect model.

This paper therefore focuses on the hard input constraints in practical batch-to-batch processes and develops a new terminal iterative learning control scheme with input saturation by considering only the terminal output tracking error. A bounded convergence is guaranteed when the initial states vary in a boundary and the desired control input exists within the saturation bound. Three cases are considered with a comparison to a standard P-type terminal ILC algorithm in the simulation study to show the efficiency of the proposed approach.

2 PROBLEM FORMULATIONS

Consider a linear continuous-time multiple input multiple output (MIMO) system, which is widely used as the linearization model of many batch processes [13],

$$\begin{cases} \dot{\mathbf{x}}_k(t) = \mathbf{A}\mathbf{x}_k(t) + \mathbf{B}\mathbf{u}_k \\ \mathbf{y}_k(t) = \mathbf{C}\mathbf{x}_k(t) \end{cases} \quad (1)$$

where $t \in [0, \dots, N]$ is the time index; N is the finite time interval of the run-to-run system; k denotes the

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system repetition number; $\mathbf{x}_k(t) \in R^p$ is the system state; $\mathbf{u}_k \in R^m$ is the system input, being constant at all sampling time in the same iteration; $\mathbf{y}_k(t) \in R^n$ is the system output, where only $\mathbf{y}_k(N)$ is measurable at the end of every run k .

From linear system theory, one can write the solution of Eq. (1) at $t = N$ as

$$\begin{aligned}\mathbf{x}_k(N) &= e^{AN} \mathbf{x}_k(0) + \left(\int_0^N e^{A(N-\tau)} B d\tau \right) \mathbf{u}_k \\ &= \bar{A} \mathbf{x}_k(0) + \bar{B} \mathbf{u}_k\end{aligned}\quad (2)$$

where $\bar{A} = e^{AN}$, $\bar{B} = \left(\int_0^N e^{A(N-\tau)} B d\tau \right)$. Thus the corresponding terminal output is

$$\mathbf{y}_k(N) = \bar{C} \bar{A} \mathbf{x}_k(0) + \bar{C} \bar{B} \mathbf{u}_k \quad (3)$$

To restrict our discussion, the following two assumptions are required.

Assumption (A1) A desired output $\mathbf{y}_d(N)$ is achievable with a unique control input $\mathbf{u}_d$ and a desired initial state $\mathbf{x}_d(0)$, i.e.,

$$\mathbf{y}_d(N) = \bar{C} \bar{A} \mathbf{x}_d(0) + \bar{C} \bar{B} \mathbf{u}_d \quad (4)$$

And the corresponding desired control input $\mathbf{u}_d$ is bounded, that is

$$\mathbf{u}_{\min} \leq \mathbf{u}_d \leq \mathbf{u}_{\max} \quad (5)$$

where $\mathbf{u}_{\min}$ and $\mathbf{u}_{\max}$ are the saturation constants.

Remark 1 Assumption A1 is a common condition in control theory and is an alternative formulation of the system controllability. It implies that the control task is solvable.

Assumption (A2) The initial state $\mathbf{x}_k(0)$ at every iteration k can be repositioned nearby the same desired starting point $\mathbf{x}_d(0)$ with some misalignment, i.e.,

$$\|\Delta \mathbf{x}_k(0)\| \leq \varepsilon_1$$

where $\Delta \mathbf{x}_k(0) = \mathbf{x}_d(0) - \mathbf{x}_k(0)$, ε_1 is a small positive constant.

Remark 2 Assumption A2 is a relaxed condition on the system initial state, which is not necessary to be strictly repetitive.

The control task is to find an optimal control input signal sequence $\mathbf{u}_k$ such that the terminal output $\mathbf{y}_k(N)$ converges to a desired terminal value $\mathbf{y}_d(N)$ at time N .

3 TERMINAL ILC DESIGN WITH INPUT SATURATION AND CONVERGENCE ANALYSIS

For the convenience of presentation, we define the following shorthand notations

$$\Delta \mathbf{y}_k = \mathbf{y}_d(N) - \mathbf{y}_k(N); \quad \Delta \mathbf{u}_k = \mathbf{u}_d - \mathbf{u}_k;$$

$$\Delta \mathbf{v}_k = \mathbf{u}_d - \mathbf{v}_k.$$

The terminal ILC scheme is developed as follows:

$$\mathbf{v}_k = \mathbf{u}_{k-1} + L \Delta \mathbf{y}_{k-1} \quad (6)$$

$$\mathbf{u}_k = \text{sat}(\mathbf{v}_k) = \begin{cases} \mathbf{u}_{\max} & \text{if } \mathbf{v}_k > \mathbf{u}_{\max} \\ \mathbf{v}_k & \text{if } \mathbf{u}_{\min} \leq \mathbf{v}_k \leq \mathbf{u}_{\max} \\ \mathbf{u}_{\min} & \text{if } \mathbf{v}_k < \mathbf{u}_{\min} \end{cases} \quad (7)$$

where L is the learning gain matrix.

Remark 3 There is no any information of $\mathbf{u}_d$ used in the design and implementation of control scheme Eqs. (6) and (7).

The validity of the above proposed terminal ILC scheme is verified by the following theorem.

Theorem 1 Consider system (1) under Assumptions A1 and A2, the terminal ILC Schemes (6), (7) guarantees that the terminal tracking error $\Delta \mathbf{y}_k$ converges to a bound, if

$$\|I - LC\bar{B}\| \leq \rho < 1 \quad (8)$$

and the convergence bound is $\|\Delta \mathbf{y}_k\| \leq \varepsilon^*$ with $\varepsilon^* =$

$$\left(\|C\bar{A}\| + \frac{\|LC\bar{A}\| \|C\bar{B}\|}{1 - \rho} \right) \varepsilon_1.$$

Proof Taking the difference between Eqs. (4) and (3) yields

$$\Delta \mathbf{y}_k = \bar{C} \bar{A} \Delta \mathbf{x}_k(0) + \bar{C} \bar{B} \Delta \mathbf{u}_k \quad (9)$$

From the control Schemes (6) and (7) we have the following control input error equation:

$$\begin{aligned}\Delta \mathbf{v}_k &= \mathbf{u}_d - \mathbf{u}_{k-1} - L \Delta \mathbf{y}_{k-1} = \Delta \mathbf{u}_{k-1} - L \Delta \mathbf{y}_{k-1} \\ &= \Delta \mathbf{u}_{k-1} - LC\bar{A} \Delta \mathbf{x}_{k-1}(0) - LC\bar{B} \Delta \mathbf{u}_{k-1} \\ &= (I - LC\bar{B}) \Delta \mathbf{u}_{k-1} - LC\bar{A} \Delta \mathbf{x}_{k-1}(0)\end{aligned}\quad (10)$$

Taking norms on both sides of Eq. (10), and using Eq. (8) and Assumption A2,

$$\|\Delta \mathbf{v}_k\| \leq \rho \|\Delta \mathbf{u}_{k-1}\| + \varepsilon_2 \quad (11)$$

is resulted with $\varepsilon_2 = \|LC\bar{A}\| \varepsilon_1$.

In terms of Eq. (5), the saturation feature in Eq. (7) leads to the following inequality:

$$\|\Delta \mathbf{u}_k\| \leq \|\Delta \mathbf{v}_k\| \quad (12)$$

Using Eqs. (11) and (12), we have

$$\|\Delta \mathbf{u}_k\| \leq \rho \|\Delta \mathbf{u}_{k-1}\| + \varepsilon_2 \quad (13)$$

According to Eqs. (8) and (12), it is clear that

$$\lim_{k \rightarrow \infty} \|\Delta \mathbf{u}_k\| \leq \frac{\varepsilon_2}{1 - \rho} \quad (14)$$

As a direct result, we have

$$\begin{aligned}\lim_{k \rightarrow \infty} \|\Delta \mathbf{y}_k\| &\leq \|C\bar{A}\| \varepsilon_1 + \|C\bar{B}\| \frac{\varepsilon_2}{1 - \rho} \\ &= \left(\|C\bar{A}\| + \frac{\|LC\bar{A}\| \|C\bar{B}\|}{1 - \rho} \right) \varepsilon_1 = \varepsilon^*\end{aligned}\quad (15)$$

Clearly, when $\varepsilon_1 = 0$, $\lim_{k \rightarrow \infty} \|\Delta \mathbf{y}_k\| = 0$.

Remark 4 It is noted that the proposed terminal ILC, Eqs. (6) and (7), is a rough-model based approach.

There is little model information required to gain a proper learning gain. In the practical implementation of the proposed approach, if the plant model is known exactly, one can give its linearization expression at the operation points, and then obtain the learning gain according to Eq. (8). If the plant model is unavailable, one can select the learning gain by “trial and error” method.

4 SIMULATION RESULTS

Consider a typical nonlinear batch reactor with reactor temperature as the control variable [22, 23]. The reaction scheme is $P_1 \xrightarrow{k_1} P_2 \xrightarrow{k_2} P_3$, where P_1 is the raw material, P_2 is the product, and P_3 is the by-product. The following differential equations describe the reactor:

$$\begin{aligned}\dot{x}_1 &= -k_1 \exp\left(\frac{-E_1}{uT_{\text{ref}}}\right) x_1^2 \\ \dot{x}_2 &= -k_1 \exp\left(\frac{-E_1}{uT_{\text{ref}}}\right) x_1^2 - k_2 \exp\left(\frac{-E_2}{uT_{\text{ref}}}\right) x_2 \quad (16) \\ y &= x_2\end{aligned}$$

where x_1 and x_2 represent the dimensionless concentrations of the raw material and the product respectively; $u = T/T_{\text{ref}}$ is the dimensionless temperature of the reactor and T_{ref} is the reference temperature; the reactor temperature T is constrained to $298 \text{ K} \leq T \leq 398 \text{ K}$; the final time t_f is fixed to be 1.0 h, the sampling time $h = t_f/N = 0.1$ with $N = 10$. The plant parameters are given as $k_1 = 4.0 \times 10^3$, $k_2 = 6.2 \times 10^5$, $E_1 = 2.5 \times 10^3 \text{ K}$, $E_2 = 5.0 \times 10^3 \text{ K}$, and $T_{\text{ref}} = 348 \text{ K}$. The initial conditions are $x_1(0) = 1$ and $x_2(0) = 0$. The operation objective of the reactor is to maximize the amount of the product (P_2) after a fixed reaction time. The desired product reference trajectory $y_d(t_f) = 0.610$ was selected from the literature [23].

In order to explore the performance of the proposed TILC to address for input constraints, three scenarios

are considered in the simulation by setting different initial control input values. For the comparison purpose, a traditional P-type terminal ILC algorithm is adopted in the following simulation:

$$u_k = u_{k-1} + L \Delta y_{k-1} \quad (17)$$

where L is the learning gain matrix, the same as that defined in Eq. (6).

In all the following cases, $L = 0.6$ is selected by “trial and error” method for both the proposed anticipatory TILC Eqs. (6), (7) and the standard P-type TILC Eq. (17).

Case 1 The control input of the first run is set as 0.97

When using the two TILC Schemes (6), (7) and Eq. (17) respectively, both the control input profiles are located in the constraint scope, i.e., $0.85632 \leq u \leq 1.14368$. Hence the same convergence property is achieved. The simulation results are shown in Fig. 1.

Case 2 The control input of the first run is set as 0.0001

Applying the proposed anticipatory TILC and standard P-type TILC, respectively, the simulation results are shown in Fig. 2. One can find that the control input [solid line in Fig. 2 (b)] is restricted into the constrained scope in the second run immediately with the saturate algorithm Eq. (7) of the anticipatory TILC, while the control input [dash line in Fig. 2 (b)] is beyond the constraint scope in several initial runs when using the standard P-type TILC Eq. (17). So the tracking performance of anticipatory TILC is better than the standard P-type TILC [Fig. 2 (a)].

Case 3 The control input of the first run is set as 1.8

Applying the proposed anticipatory TILC, the simulation results are shown in Fig. 3. Although the escape phenomenon of the terminal tracking error occurs at the first run [Fig. 3 (a)], the terminal output still converges to the desired value and the control input signal is bounded in the constraint scope [Fig. 3 (b)].

While applying the standard P-type TILC Eq. (17), both the system output and control input are divergent to infinity and the data escape phenomenon occurs for all iterations.

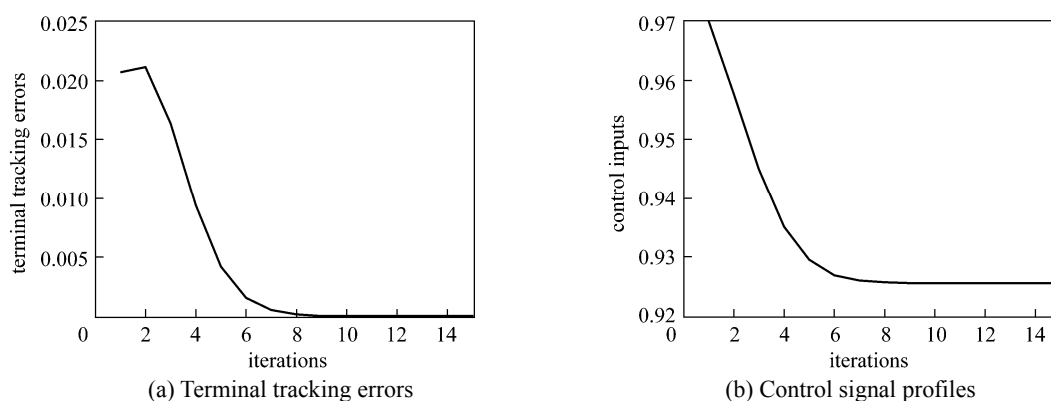


Figure 1 The same control performance of anticipatory TILC and standard P-type TILC with $u_1 = 0.97$

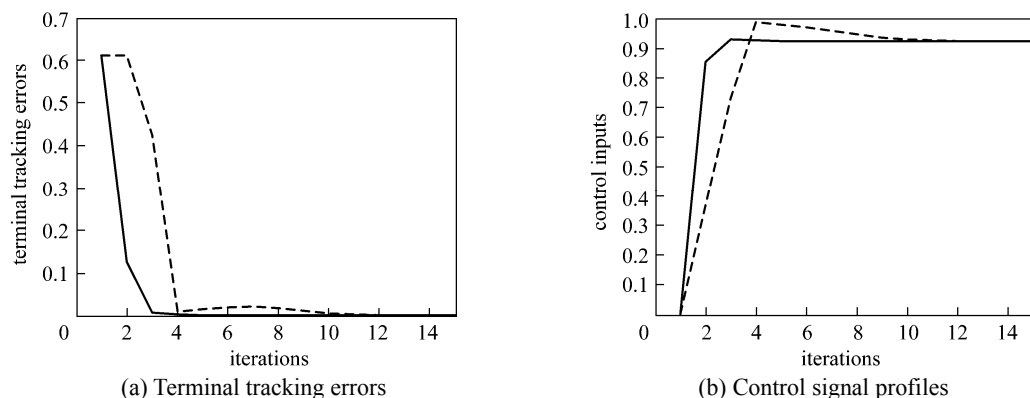


Figure 2 Compared results of anticipatory TILC and standard P-type TILC with $u_1 = 0.0001$

— anticipatory TILC; - - - standard P-type TILC

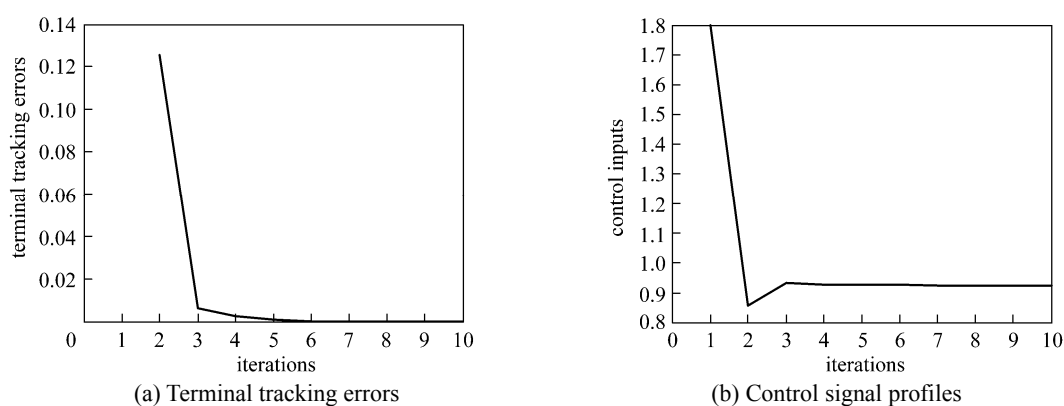


Figure 3 The control performance of the proposed anticipatory TILC with $u_1 = 1.8$

5 CONCLUSIONS

In this paper, a batch-to-batch process with input constraints is considered and an anticipatory terminal iterative learning control is proposed, where only the terminal output tracking error, rather than the whole output trajectory, is used to updating the control input signal iteratively. Both convergence analysis and simulation results show the effectiveness and applicability of the proposed TILC approach.

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